to learn as much as possible to extract as much value as possible from each problem. In contrast, agents in layer $L-1$ are less willing to learn, since they can sell their problems to agents in layer L. Agents in layer $L-2$ get a higher price for their unsolved problems, so their incentives to learn are smaller than those of the agents above them. Again, this logic applies to all layers in the hierarchy, including layer 0 where the fall in knowledge is even larger, since workers can span their knowledge over only one problem instead of $1/h$ of them (since they use their time to produce). Of course, as $L \to \infty$ there is no final layer and so this logic does not apply, and all prices and knowledge levels of problem solvers are constant, since there is no final layer in which prices are equal to zero.

To prove the next proposition we will use the following parameter restriction, which is necessary and sufficient for $z_L^\ell > 0$ for all ℓ and L.

Condition 1. $A \geqslant 1$, $h < 1$ and A, λ, $\bar{c}$ and h satisfy

$$\frac{A\lambda}{\bar{c}} > \frac{1}{h} + \ln h.$$

Proposition 2. *Under Condition* 1, *for any A and L finite, there exists a unique equilibrium determined by a set of prices $\{r_L^\ell\}_{\ell=0}^{L-1}$ and a set of knowledge levels $\{z_L^\ell\}_{\ell=0}^{L}$ such that $r_L^\ell > 0$ is strictly decreasing in ℓ and $z_L^\ell > 0$ is strictly increasing in ℓ.*

Proof. See Appendix A. □

We now turn to the properties of this economy as we change the highest layer L. Note that, for now, without radical innovations, changes in L happen as time evolves, and so studying the properties of our economy as we change the number of layers is equivalent to studying the properties of our economy as time evolves. This equivalence will change in the next section, once we introduce radical innovations. The next proposition shows that as the number of layers increases, so do wages (or output per capita if knowledge costs are considered forgone output). Furthermore, since wages are bounded by w_∞, there are eventual decreasing returns in the number of organizational layers. This is just the result of higher layers dealing with fewer problems, since they are more rare. So adding an extra layer contributes to output per capita (since more problems are solved), but it contributes less the higher the layer, since there are fewer and fewer problems that require such specialized knowledge.

The proposition also shows that as time evolves and the number of layers increases, r_L^ℓ increases and z_L^ℓ decreases for all ℓ. The first result is a direct consequence of the logic used in the previous proposition. As time elapses and the number of layers increases, the number of layers above a given ℓ increases, which implies that r_L^ℓ increases, since the problems can be resold further if not solved. In turn, higher prices imply less knowledge acquisition, since the opportunity to resell problems is a substitute for solving them.

Proposition 3. *Under Condition* 1, *for any technology A, as the number of layers L increases, w_I increases and $\lim_{L\to\infty} \bar{w}(A,L) = w_\infty$. Furthermore, as the number of layers L increases, prices r_L^ℓ increase for all $\ell = 0,\ldots,L-1$ and knowledge levels z_L^ℓ decrease for all $\ell = 0,\ldots,L$. As $L \to \infty$, $r_L^\ell \to r_\infty$ for all $\ell = 0,\ldots,L-1$ and $z_L^\ell \to z_\infty^0$ all $\ell = 0,\ldots,L$.*

Proof. See Appendix A. □