LING 230 **Handout: Basic Rules of Natural Deduction (from Gamut)** **Fall 2009**

Monday, October 5th

I∧ introduction rule for ∧

$\phi, \psi \vdash \phi \wedge \psi$ and
$\phi, \psi \vdash \psi \wedge \phi$ (p.129)

i	ϕ	
	$\vdots$	
j	ψ	
	$\vdots$	
k	$\phi \wedge \psi$	I∧, i, j
k'	$\psi \wedge \phi$	I∧, j, i

E∧ elimination rule for ∧

$\phi \wedge \psi \vdash \phi$ and
$\phi \wedge \psi \vdash \psi$ (p.130)

i	$\phi \wedge \psi$	
	$\vdots$	
j	ϕ	E∧, i
j'	ψ	E∧, i

I∨ introduction rule for ∨

$\phi \vdash \phi \vee \psi$ and
$\phi \vdash \psi \vee \phi$ (p.135–136)

i	ϕ	
	$\vdots$	
j	$\phi \vee \psi$	I∨, i
j'	$\psi \vee \phi$	I∨, i

E∨ elimination rule for ∨

$\phi \vee \psi, \phi \to \chi, \psi \to \chi \vdash \chi$ (p.136)

i	$\phi \vee \psi$	
	$\vdots$	
j	$\phi \to \chi$	
	$\vdots$	
k	$\psi \to \chi$	
	$\vdots$	
ℓ	χ	E∨, i, j, k

E→ elimination rule for →,

a.k.a. **MP** *modus ponens*

$\phi \to \psi, \phi \vdash \psi$ (p.131)

i	$\phi \to \psi$	
	$\vdots$	
j	ϕ	
	$\vdots$	
k	ψ	E→ (or MP), i, j

Rep repetition rule

$\phi \vdash \phi$ (p.135)

i	ϕ	
	$\vdots$	
j	ϕ	Rep, i

E¬ elimination rule for ¬

$\neg\phi, \phi \vdash \bot$ (p.137–138)

i	$\neg\phi$	
	$\vdots$	
j	ϕ	
	$\vdots$	
k	$\bot$	E¬, i, j

EFSQ *ex falso sequitur quodlibet*

$\bot \vdash \phi$ (p.139)

i	$\bot$	
	$\vdots$	
j	ϕ	EFSQ, i

¬¬ double negation rule

$\neg\neg\phi \vdash \phi$ (p.140)

i	$\neg\neg\phi$	
	$\vdots$	
j	ϕ	¬¬, i

(not in Gamut)

I↔ introduction rule for $\leftrightarrow$

$\phi \to \psi, \psi \to \phi \vdash \phi \leftrightarrow \psi$ and
$\phi \to \psi, \psi \to \phi \vdash \psi \leftrightarrow \phi$

i	$\phi \to \psi$	
	$\vdots$	
j	$\psi \to \phi$	
	$\vdots$	
k	$\phi \leftrightarrow \psi$	I↔, i, j
k'	$\psi \leftrightarrow \phi$	I↔, j, i

HS hypothetical syllogism

$\phi \to \psi, \psi \to \chi \vdash \phi \to \chi$

i	$\phi \to \psi$	
	$\vdots$	
j	$\psi \to \chi$	
	$\vdots$	
k	$\phi \to \chi$	HS, i, j

E↔ elimination rule for $\leftrightarrow$

$\phi \leftrightarrow \psi \vdash \phi \to \psi$ and
$\phi \leftrightarrow \psi \vdash \psi \to \phi$

i	$\phi \leftrightarrow \psi$	
	$\vdots$	
j	$\phi \to \psi$	E↔, i
j'	$\psi \to \phi$	E↔, i

CD constructive dilemma

$\phi \vee \psi, \phi \to \chi, \psi \to \xi \vdash \chi \vee \xi$

i	$\phi \vee \psi$	
	$\vdots$	
j	$\phi \to \chi$	
	$\vdots$	
k	$\psi \to \xi$	
	$\vdots$	
ℓ	$\chi \vee \xi$	CD, i, j, k

DS disjunctive syllogism

$\phi \vee \psi, \neg\phi \vdash \psi$ and
$\psi \vee \phi, \neg\phi \vdash \psi$

i	$\phi \vee \psi$	
i'	$\psi \vee \phi$	
	$\vdots$	
j	$\neg\phi$	
	$\vdots$	
k	ψ	DS, i, j
k'	ψ	DS, i', j

MT *modus tollens*

$\phi \to \psi, \neg\psi \vdash \neg\phi$

i	$\phi \to \psi$	
	$\vdots$	
j	$\neg\psi$	
	$\vdots$	
k	$\neg\phi$	MT, i, j