

**Problem 1: Truth Tables.**

a. 

A	B	C	$A \rightarrow \neg(B \wedge C)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

b. 

P	Q	R	$(P \leftrightarrow \neg Q) \leftrightarrow (R \vee Q)$
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

c. 

X	Y	Z	$(X \wedge (\neg(\neg Y \vee X) \leftrightarrow Z)) \rightarrow Y$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

**Problem 2: Equivalence.**

a. 

P	Q	$P \Delta Q$
1	1	0
1	0	1
0	1	1
0	0	1

b.  $\neg P$  is logically equivalent to  $P \Delta P$ .

c.  $P \wedge Q$  is logically equivalent to  $(P \Delta Q) \Delta (P \Delta Q)$ .

P	Q	$(P \Delta Q) \Delta (P \Delta Q)$
1	1	0
1	0	1
0	1	1
0	0	1

d.  $P \vee Q$  is logically equivalent to  $(P \Delta P) \Delta (Q \Delta Q)$ .

P	Q	$(P \Delta P) \Delta (Q \Delta Q)$
1	1	0
1	0	1
0	1	1
0	0	1

e.  $P \rightarrow Q$  is logically equivalent to  $((P \Delta P) \Delta (P \Delta P)) \Delta (Q \Delta Q)$

P	Q	$((P \Delta P) \Delta (P \Delta P)) \Delta (Q \Delta Q)$
1	1	0
1	0	0
0	1	1
0	0	1

$P \rightarrow Q$  is also logically equivalent to the much shorter  $(P \Delta Q) \Delta P$

P	Q	$(P \Delta Q) \Delta P$
1	1	0
1	0	1
0	1	1
0	0	1

As for  $P \leftrightarrow Q$ , it is logically equivalent to both of these monstrous sentences:

$$\left( \left( \left( (P \Delta P) \Delta (Q \Delta Q) \right) \Delta \left( (P \Delta P) \Delta (Q \Delta Q) \right) \right) \Delta \left( \left( (P \Delta P) \Delta (Q \Delta Q) \right) \Delta \left( (P \Delta P) \Delta (Q \Delta Q) \right) \right) \right) \Delta \left( \left( (P \Delta Q) \Delta (P \Delta Q) \right) \Delta \left( (P \Delta Q) \Delta (P \Delta Q) \right) \right)$$

$$\left( \left( \left( (P \Delta P) \Delta (P \Delta P) \right) \Delta (Q \Delta Q) \right) \Delta \left( \left( (Q \Delta Q) \Delta (Q \Delta Q) \right) \Delta (P \Delta P) \right) \right) \Delta \left( \left( \left( (P \Delta P) \Delta (P \Delta P) \right) \Delta (Q \Delta Q) \right) \Delta \left( \left( (Q \Delta Q) \Delta (Q \Delta Q) \right) \Delta (P \Delta P) \right) \right)$$

But using the shorter rewrite for  $P \rightarrow Q$ , we can rewrite  $P \leftrightarrow Q$  with only 11 triangles!

$$\left( \left( (P \Delta Q) \Delta P \right) \Delta \left( (Q \Delta P) \Delta Q \right) \right) \Delta \left( \left( (P \Delta Q) \Delta P \right) \Delta \left( (Q \Delta P) \Delta Q \right) \right)$$

**Problem 3: Translation.**

- a.  $X = \neg S \vee G$   
 $Y = \neg T \rightarrow S$   
 $Z = T \rightarrow \neg G$

	$S$	$G$	$T$	$\neg S$	$\vee$	$G$	$\neg T$	$\rightarrow$	$S$	$T$	$\rightarrow$	$\neg G$	
$V_1$	1	1	1	0	1	1	0	1	1	1	0	0	
$V_2$	1	1	0	0	1	1	1	1	1	0	1	0	$\Leftarrow V_2(X) = V_2(Y) = V_2(Z) = 1$
$V_3$	1	0	1	0	0	0	0	1	1	1	1	1	
$V_4$	1	0	0	0	0	0	1	1	1	1	1	1	
$V_5$	0	1	1	1	1	1	0	1	1	0	0	0	
$V_6$	0	1	0	1	1	1	1	0	1	1	1	0	
$V_7$	0	0	1	1	1	1	0	1	1	1	1	1	$\Leftarrow V_7(X) = V_7(Y) = V_7(Z) = 1$
$V_8$	0	0	0	1	1	1	1	0	1	1	1	1	

b.  $X$ ,  $Y$ , and  $Z$  are all true at the same time only in valuations  $V_2$  and  $V_7$ . In each of the other valuations, at least one of  $X$ ,  $Y$ , or  $Z$  is false. The truth values of  $S$  in these two valuations are  $V_2(S) = 1$  and  $V_7(S) = 0$ . Since Sharpay's scheme was unsuccessful,  $V(S) = 0$  (which is equivalent to  $V(\neg S) = 1$ ). This means that only valuation  $V_7$  satisfies the required conditions, and in this valuation,  $V_7(G) = 0$  and  $V_7(T) = 1$ . So, if we know that  $X$ ,  $Y$ , and  $Z$  are all true and that  $S$  is false, then we also know that Gabriella was not given a gift, and Troy did tap-dance with his team.

c. The sentence  $\neg S \wedge (\neg G \wedge T)$  satisfies the required conditions. It contains exactly one instance of each of the atomic sentences  $S$ ,  $G$ , and  $T$  (and no others), it is not a tautology because there are valuations in which it is false (for example, valuation  $V_1$ ), and it is logically consistent with  $X$ ,  $Y$ ,  $Z$ , and  $\neg S$ , because there is a valuation in which all of these five sentences are true (valuation  $V_7$ ).

	$S$	$G$	$T$	$\neg S$	$\wedge$	$(\neg G \wedge T)$
$V_1$	1	1	1	0	0	0 0
$V_2$	1	1	0	0	0	0 0
$V_3$	1	0	1	0	0	1 1
$V_4$	1	0	0	0	0	1 0
$V_5$	0	1	1	1	0	0 0
$V_6$	0	1	0	1	0	0 0
$V_7$	0	0	1	1	1	1 1
$V_8$	0	0	0	1	0	1 0

Other sentences satisfying these conditions include various permutations like  $(\neg S \wedge \neg G) \wedge T$  and  $\neg G \wedge (\neg S \wedge T)$ , as well as more complex sentences like  $\neg((S \vee G) \vee \neg T)$ .