LING 230 **Homework #1: Propositional Logic** **Fall 2009**

due Monday, September 21st

Problem 1: Truth Tables. For the following sentences of propositional logic, give full, detailed truth tables with columns of truth values for each connective, circling the column for the main connective, as in the sample to the right:

a. $A \to \neg(B \wedge C)$
b. $(P \leftrightarrow \neg Q) \leftrightarrow (R \vee Q)$
c. $\Big(X \wedge \big(\neg(\neg Y \vee X) \leftrightarrow Z\big)\Big) \to Y$

	J	K	$(J \wedge K)$	$\vee$	$\neg$	$(K \leftrightarrow J)$
V_1	1	1	1	1	0	1
V_2	1	0	0	1	1	0
V_3	0	1	0	1	1	0
V_4	0	0	0	0	0	1

Problem 2: Logical Equivalence. Five logical connectives is more than enough for a complete logical system. We can in fact get by with just a single two-place logical connective to express the same range of truth values that can be expressed with $\neg$, $\wedge$, $\vee$, $\to$, and $\leftrightarrow$. Let $\triangle$ be a two-place connective such that $P \triangle Q$ is false if both P and Q are true, and $P \triangle Q$ is true otherwise.

a. Give a truth table for $P \triangle Q$.

b. Show that $\neg$ is unnecessary in a system with $\triangle$ by coming up with a new sentence that is logically equivalent to $\neg P$, using $\triangle$ as the only logical connective.

c. Do the same for $P \wedge Q$. This can be accomplished with as few as three instances of $\triangle$ in your new sentence (but you can use more if necessary). Give a full, detailed truth table for your new sentence to confirm that it is equivalent to $P \wedge Q$.

d. Do the same for $P \vee Q$, including a truth table. You'll again need at least three instances of $\triangle$.

e. Do the same for $P \to Q$, including a truth table. You may find it helpful to first come up with a new sentence that is logically equivalent to $P \to Q$, using $\neg$ and $\vee$ as the only logical connectives. Then create another new sentence that is logically equivalent to this intermediate sentence, this time using $\triangle$ as the only logical connective, based on your work in Problems 2b and 2d. You'll need at least five instances of $\triangle$ in your final sentence.

[If you're ambitious, try coming up with a sentence that is logically equivalent to $P \leftrightarrow Q$, using $\triangle$ as the only logical connective. I can't do it with fewer than 23(!) instances of $\triangle$. Can you do better?]

Problem 3: Translation. Let S, G, and T be atomic sentences such that $S =$ 'Sharpay's scheme was successful', $G =$ 'Gabriella was given a gift', and $T =$ 'Troy tap-danced with his team'.

a. Translate the following sentences to complex sentences in propositional logic, as directly and plainly as possible, using only the atomic sentences S, G, and T. Give a full, detailed truth table showing the truth values of your translations of X, Y, and Z.

$X =$ 'Either Sharpay's scheme failed, or someone gave Gabriella a gift (or perhaps both)'
$Y =$ 'If Troy didn't tap-dance with his team, then Sharpay's scheme succeeded'
$Z =$ 'No one gave Gabriella a gift if Troy tap-danced with his team'

b. During the extravagant musical finale, Sharpay's scheme was revealed to be unsuccessful, as usual. If X, Y, and Z are all true, what are the truth values for G and T (i.e., was Gabriella given a gift, and did Troy tap-dance with his team)? Explain in clear, sufficient prose how to use the valuations and truth table from 3a to figure out what happened.

c. Come up with any one sentence in propositional logic that (i) contains exactly one instance of each of the atomic sentences S, G, and T (and no others), (ii) is not a tautology, and (iii) is logically consistent with the world as we know it (i.e., with X, Y, Z, and $\neg S$). Give the full, detailed truth table for this sentence, using the same valuations as in 3a.