LING 230 **Homework #2: Predicate Logic** **Fall 2009**

solutions

Problem 1: Translation.

a. $\neg Zf \wedge \neg Zd$

b. $Pc_1 \rightarrow (Pa \wedge Pe)$

c. $Zm \leftrightarrow Zc_2$

d. $\neg(Pm \vee Pr) \wedge (Pf \wedge Pc_2)$

e. $(Lda \vee Ldc_2) \wedge \neg(Lda \wedge Ldc_2)$

f. $(Lfe \wedge Lef) \wedge (Lrc_1 \wedge Lc_1 r)$

Problem 2: Valuation in a Model.

a. The required truth values impose restrictions on how I must be defined:

$V_{\mathbb{M}}(\text{1a}) = 0$ means that $I(Z)$ must contain at least one of FRANNIE and DALE.

$V_{\mathbb{M}}(\text{1b}) = 1$ means that $I(P)$ must exclude CAPPIE and/or contain both ASHLEIGH and EVAN.

$V_{\mathbb{M}}(\text{1c}) = 0$ means that $I(Z)$ must contain exactly one of MAX and CASEY.

$V_{\mathbb{M}}(\text{1d}) = 1$ means that $I(P)$ must exclude both MAX and REBECCA, and it must contain both FRANNIE and CASEY.

$V_{\mathbb{M}}(\text{1e}) = 1$ means that $I(L)$ must contain exactly one of $\langle$DALE, ASHLEIGH$\rangle$ and $\langle$DALE, CASEY$\rangle$.

$V_{\mathbb{M}}(\text{1f}) = 0$ means that $I(L)$ must exclude at least one of $\langle$FRANNIE, EVAN$\rangle$, $\langle$EVAN, FRANNIE$\rangle$, $\langle$REBECCA, CAPPIE$\rangle$, and $\langle$CAPPIE, REBECCA$\rangle$.

There are many ways to make all of these conditions true. Here is one possible solution:

$$I(P) = \left\{ \begin{array}{c} \text{ASHLEIGH}, \text{CAPPIE}, \text{CASEY}, \\ \text{EVAN}, \text{FRANNIE} \end{array} \right\} \qquad I(Z) = \{\text{ASHLEIGH}, \text{CASEY}, \text{FRANNIE}, \text{REBECCA}\}$$

$$I(L) = \left\{ \begin{array}{c} \langle\text{CAPPIE}, \text{CASEY}\rangle, \langle\text{CASEY}, \text{CAPPIE}\rangle, \langle\text{CASEY}, \text{EVAN}\rangle, \langle\text{CASEY}, \text{MAX}\rangle, \\ \langle\text{DALE}, \text{CASEY}\rangle, \langle\text{EVAN}, \text{CASEY}\rangle, \langle\text{EVAN}, \text{FRANNIE}\rangle, \langle\text{EVAN}, \text{REBECCA}\rangle, \\ \langle\text{FRANNIE}, \text{EVAN}\rangle, \langle\text{MAX}, \text{CASEY}\rangle, \langle\text{REBECCA}, \text{EVAN}\rangle \end{array} \right\}$$

b. $V_{\mathbb{M}}\big(Pc_1 \rightarrow (Pa \wedge Pe)\big) = 1$ iff $\underbrace{V_{\mathbb{M}}(Pc_1) = 0}_{(1)}$ or $\underbrace{V_{\mathbb{M}}(Pa \wedge Pe) = 1}_{(2)}$.

Consider case (2), in which $V_{\mathbb{M}}(Pa \wedge Pe) = 1$. This happens iff $V_{\mathbb{M}}(Pa) = 1$ and $V_{\mathbb{M}}(Pe) = 1$, which happens iff $I(a) \in I(P)$ and $I(e) \in I(P)$, which happens iff ASHLEIGH $\in I(P)$ and EVAN $\in I(P)$. Since ASHLEIGH and EVAN are indeed both elements of $I(P)$, case (2) is satisfied.

Only one of (1) and (2) needs to be satisfied, so $V_{\mathbb{M}}\big(Pc_1 \rightarrow (Pa \wedge Pe)\big) = 1$. (Note that case (1) is <u>not</u> satisfied, so considering it would not have helped prove our primary valuation.)

c. $V_{\mathbb{M}}\big((Lfe \wedge Lef) \wedge (Lrc_1 \wedge Lc_1 r)\big) = 0$ iff $\underbrace{V_{\mathbb{M}}(Lfe \wedge Lef) = 0}_{(1)}$ or $\underbrace{V_{\mathbb{M}}(Lrc_1 \wedge Lc_1 r) = 0}_{(2)}$.

Consider case (2), in which $V_{\mathbb{M}}(Lrc_1 \wedge Lc_1 r) = 0$. This happens iff $\underbrace{V_{\mathbb{M}}(Lrc_1) = 0}_{(2a)}$ or $\underbrace{V_{\mathbb{M}}(Lc_1 r)\big) = 0}_{(2b)}$.

Further consider case (2a), in which $V_{\mathbb{M}}(Lrc_1) = 0$. This happens iff $\langle I(r), I(c_1)\rangle \notin I(L)$, which happens iff $\langle$REBECCA, CAPPIE$\rangle \notin I(L)$. Since the pair $\langle$REBECCA, CAPPIE$\rangle$ is indeed not an element of $I(L)$, case (2a) is satisfied.

Only one of (2a) and (2b) needs to be satisfied, so case (2) is satisfied. (Note that case (2b) is also satisfied, so we could have considered it instead of (2a).)

Only one of (1) and (2) needs to be satisfied, so $V_{\mathbb{M}}\big((Lfe \wedge Lef) \wedge (Lrc_1 \wedge Lc_1 r)\big) = 0$. (Note that case (1) is <u>not</u> satisfied, so considering it would not have helped prove our primary valuation.)

Problem 3: Proof. Let *Pab* and *Pba* be WFFs of a formal language $\mathscr{L}$, and let $\mathbb{M}$ be a model of $\mathscr{L}$ with a domain of entities $\mathscr{D} = \{\textsc{Aa}, \textsc{Bb}\}$ and an interpretation function I such that $I(a) = \textsc{Aa}$ and $I(b) = \textsc{Bb}$.

Assume that P is symmetric in $\mathbb{M}$. In our logic system, any given valuation like $V_{\mathbb{M}}(Pab)$ can only have one of two values, either 1 or 0, so for this particular valuation, we have two cases which must both be considered: (1) $V_{\mathbb{M}}(Pab) = 1$ and (2) $V_{\mathbb{M}}(Pab) = 0$.

Consider case (1), in which $V_{\mathbb{M}}(Pab) = 1$. This happens iff $\langle I(a), I(b)\rangle \in I(P)$, which happens iff $\langle \textsc{Aa}, \textsc{Bb}\rangle \in I(P)$. But since P is symmetric, this also means that $\langle \textsc{Bb}, \textsc{Aa}\rangle \in I(P)$, which happens iff $\langle I(b), I(a)\rangle \in I(P)$, which happens iff $V_{\mathbb{M}}(Pba) = 1$. Since both $V_{\mathbb{M}}(Pab)$ and $V_{\mathbb{M}}(Pba)$ are equal to the same number, they are equal to each other, so $V_{\mathbb{M}}(Pab) = V_{\mathbb{M}}(Pba)$.

Consider case (2), in which $V_{\mathbb{M}}(Pab) = 0$. This happens iff $\langle I(a), I(b)\rangle \notin I(P)$, which happens iff $\langle \textsc{Aa}, \textsc{Bb}\rangle \notin I(P)$. But since P is symmetric, this also means that $\langle \textsc{Bb}, \textsc{Aa}\rangle \notin I(P)$, which happens iff $\langle I(b), I(a)\rangle \notin I(P)$, which happens iff $V_{\mathbb{M}}(Pba) = 0$. Since both $V_{\mathbb{M}}(Pab)$ and $V_{\mathbb{M}}(Pba)$ are equal to the same number, they are equal to each other, so $V_{\mathbb{M}}(Pab) = V_{\mathbb{M}}(Pba)$.

Thus, in both possible cases stemming from our assumption, it is necessary for $V_{\mathbb{M}}(Pab) = V_{\mathbb{M}}(Pba)$. By the definition of the valuation of material equivalence, this means that $V_{\mathbb{M}}(Pab \leftrightarrow Pba) = 1$.