**LING 230** **Homework #2: Predicate Logic** **Fall 2009**

due Monday, September 28th

**Problem 1: Translation.** Let $\mathscr{L}$ be a formal language containing the individual constants $a$, $c_1$, $c_2$, $d$, $e$, $f$, $m$, and $r$, and the predicates $H$, $J$, and $L$, with the following key for the translation between $\mathscr{L}$ and English:

| | | |
|---|---|---|
| $a$ : *Ashleigh* | $e$ : *Evan* | $Px$ : *x was (ever) president of a house* |
| $c_1$ : *Cappie* | $f$ : *Frannie* | $Zx$ : *x was (ever) a member of ZBZ* |
| $c_2$ : *Casey* | $m$ : *Max* | $Lxy$ : *x fell in love with y* |
| $d$ : *Dale* | $r$ : *Rebecca* | |

Translate the following English sentences to WFFs of $\mathscr{L}$ as directly and plainly as possible. Remember that not all subtleties of meaning present in an English sentence can be represented yet in our formal logical system.

a. Frannie was never a member of ZBZ, and neither was Dale.
b. Ashleigh and Evan were each president of a house if Cappie ever was.
c. Max was a member of ZBZ if and only if Casey was, too.
d. Neither Max nor Rebecca were ever president of a house, although Frannie and Casey both were.
e. Dale fell in love with exactly one of Ashleigh and Casey.
f. Frannie and Evan fell in love with each other, as did Rebecca and Cappie.

**Problem 2: Valuation in a Model.** Assume $\mathscr{L}$ as in Problem 1. Further assume a model $\mathbb{M}$ of $\mathscr{L}$ consisting of a domain of entities $\mathscr{D}$ (defined below) and an interpretation function $I$ (partly defined below).

$$\mathscr{D} = \left\{ \begin{array}{c} \text{Ashleigh}, \text{Cappie}, \text{Casey}, \\ \text{Dale}, \text{Evan}, \text{Frannie}, \\ \text{Max}, \text{Rebecca} \end{array} \right\}$$

$$\begin{array}{rclrcl} I(a) & = & \text{Ashleigh} & I(e) & = & \text{Evan} \\ I(c_1) & = & \text{Cappie} & I(f) & = & \text{Frannie} \\ I(c_2) & = & \text{Casey} & I(m) & = & \text{Max} \\ I(d) & = & \text{Dale} & I(r) & = & \text{Rebecca} \end{array}$$

a. Finish defining $\mathbb{M}$ by providing definitions of $I(P)$, $I(Z)$, and $I(L)$ so that sentences 1b, d, and e are all true in $\mathbb{M}$, and 1a, c, and f are all false in $\mathbb{M}$. There is more than one possible solution; you can just use the simplest one, if you prefer.

b. Show explicitly, step-by-step, how to calculate $V_{\mathbb{M}}(1\text{b}) = 1$.

c. Show explicitly, step-by-step, how to calculate $V_{\mathbb{M}}(1\text{f}) = 0$.

**Problem 3: Proof.** Symmetric predicates are two-place predicates like *next to*, *married to*, and *touched*, in which the two participants in the predicate can be swapped without changing the truth value. For example, if *Alice is next to Bob* is true, then we expect *Bob is next to Alice* to also be true. It is simply impossible given our usual interpretation of *next to* for two people to be arranged in such a way that one is next to the other, but not vice versa. We will formally define a symmetric predicate as follows:

> **Definition:** Let $P$ be a two-place predicate in a formal language $\mathscr{L}$, and let $\mathbb{M}$ be a model with a domain of entities $\mathscr{D}$ and an interpretation function $I$. Then $P$ is symmetric in $\mathbb{M}$ iff for any arbitrary entities $\varepsilon_1, \varepsilon_2 \in \mathscr{D}$, if $\langle \varepsilon_1, \varepsilon_2 \rangle \in I(P)$, then $\langle \varepsilon_2, \varepsilon_1 \rangle \in I(P)$.

Let $Pab$ and $Pba$ be WFFs of a formal language $\mathscr{L}$, and let $\mathbb{M}$ be a model of $\mathscr{L}$ with a domain of entities $\mathscr{D} = \{\text{Aa}, \text{Bb}\}$ and an interpretation function $I$ such that $I(a) = \text{Aa}$ and $I(b) = \text{Bb}$. Prove that if $P$ is symmetric in $\mathbb{M}$, then $V_{\mathbb{M}}(Pab \leftrightarrow Pba) = 1$.