LING 230 **Homework #3: Quantification in Predicate Logic** **Fall 2009**

due Monday, October 5th

Problem 1: Translation. Let $\mathscr{L}$ be a formal language containing the individual constants *a*, *b*, *d*, *g*, *h*, *r*, *s*, and *t* and the predicates *C*, *H*, *K*, and *L*, with the following key for the translation between $\mathscr{L}$ and English:

a : *Apollo*	*h* : *Helo*	*Cx* : *x is a Cylon*
b : *Boomer*	*r* : *Rosalyn*	*Hx* : *x is a human*
d : *D'Anna*	*s* : *Six*	*Kxy* : *x tried to kill y*
g : *Gaius*	*t* : *Tigh*	*Lxy* : *x loves y*

Translate the following English sentences to WFFs of $\mathscr{L}$ as directly and plainly as possible. Remember that not all subtleties of meaning present in an English sentence can be represented yet in our formal logical system.

a. Some Cylon loves a human.
b. Everyone who isn't a human is a Cylon.
c. No Cylon that loves Helo tried to kill him.
d. If Tigh tried to kill a human, then everyone Tigh loves is a Cylon.
e. D'Anna, Boomer, and Six are all Cylons who have tried to kill each other.
f. Anyone Apollo has tried to kill was a Cylon.
g. Gaius is a human that loves at least two Cylons.
h. Rosalyn tried to kill someone she loves.
i. Rosalyn didn't try to kill anyone she doesn't love.
j. There is a human that every Cylon tried to kill.

Problem 2: Valuation in a Model. Assume $\mathscr{L}$ as in Problem 1. Further assume a model $\mathbb{M}$ of $\mathscr{L}$ consisting of a domain of entities $\mathscr{D}$ (defined below) and an interpretation function *I* (partly defined below).

$$\mathscr{D} = \left\{ \begin{array}{c} \text{APOLLO}, \text{BOOMER}, \\ \text{D'ANNA}, \text{GAIUS}, \text{HELO}, \\ \text{ROSALYN}, \text{SIX}, \text{TIGH} \end{array} \right\}$$

$$\begin{array}{llll} I(a) = \text{APOLLO} & & I(h) = \text{HELO} \\ I(b) = \text{BOOMER} & & I(r) = \text{ROSALYN} \\ I(d) = \text{D'ANNA} & & I(s) = \text{SIX} \\ I(g) = \text{GAIUS} & & I(t) = \text{TIGH} \end{array}$$

a. Finish defining $\mathbb{M}$ by providing definitions of $I(C)$, $I(H)$, $I(K)$, and $I(L)$ so that sentences 1a, b, d, e, g, and j are all true in $\mathbb{M}$, and 1c, f, h, and i are all false in $\mathbb{M}$.

b. Show explicitly, step-by-step, how to calculate $V_{\mathbb{M}}(1a) = 1$.

c. Show explicitly, step-by-step, how to calculate $V_{\mathbb{M}}(1c) = 0$.

Problem 3: Proof. Let $\forall x(Px \leftrightarrow Qx)$ be a WFF of a formal language $\mathscr{L}$, and let $\mathbb{M}$ be a model of $\mathscr{L}$ with a domain of entities $\mathscr{D}$ and an interpretation function *I*. Prove that if $I(P) = I(Q)$ then $V_{\mathbb{M}}\big(\forall x(Px \leftrightarrow Qx)\big) = 1$.