

For the logical arguments in Problems 1 and 2, you may use any of the following rules, but no others:

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|-----------|---------|-------------------|---------|------------|--------------------|----|----|
| $I\wedge$ | $I\vee$ | $E\rightarrow/MP$ | $E\neg$ | $\neg\neg$ | $I\leftrightarrow$ | DS | CD |
| $E\wedge$ | $E\vee$ | Rep | EFSQ | | $E\leftrightarrow$ | HS | MT |

Problem 1: Short logical arguments. Prove the following statements with logical arguments using any of the rules of natural deduction allowed for this homework. These arguments can each be done in three to six lines each, but more is okay.

- $X \leftrightarrow Y \vdash X \leftrightarrow X$
- $P \wedge Q \vdash Q \vee P$
- $J \rightarrow K, \neg K \vdash \neg J \wedge \neg K$
- $(Qbc \wedge Qca) \wedge \neg Qbc \vdash \neg Qac$
- $\neg \exists x Aax, \neg \forall y Eye \vee \exists x Aax \vdash \neg \forall y Eye$

Problem 2: Longer logical argument. Prove the following statement with a logical argument using any of the rules of natural deduction allowed for this homework. This argument can be done in fifteen lines, but more is okay.

$$\neg A \wedge B, C \rightarrow \neg B, \neg C \leftrightarrow D, (E \vee B) \rightarrow (\neg D \vee A) \vdash A \vee \neg B$$

Problem 3: Proving semantic validity. Show that disjunctive syllogism (DS), modus tollens (MT), and the elimination rule for \vee ($E\vee$) are semantically valid. That is, prove each of the following statements:

- $\phi \vee \psi, \neg \psi \models \phi$ (DS)
- $\phi \rightarrow \psi, \neg \psi \models \neg \phi$ (MT)
- $\phi \vee \psi, \phi \rightarrow \chi, \psi \rightarrow \chi \models \chi$ ($E\vee$)