

## Problem 1: Short logical arguments.

a. 1	$P \rightarrow \neg Q$	assumption
2	$Q$	assumption
3	$P$	assumption
4	$\neg Q$	$E\rightarrow/MP, 1, 3$
5	$\perp$	$E\neg, 4, 2$
6	$\neg P$	$I\neg/RAA$

b. 1	$X \vee \neg Y$	assumption
2	$\neg Y \rightarrow X$	assumption
3	$X$	assumption
4	$X$	Rep, 3
5	$X \rightarrow X$	$I\rightarrow/CP$
6	$X$	$E\vee, 1, 5, 2$

c. 1	$\neg(A \wedge B)$	assumption
2	$B$	assumption
3	$A$	assumption
4	$A \wedge B$	$I\wedge, 3, 2$
5	$\perp$	$E\neg, 1, 4$
6	$\neg A$	$I\neg/RAA$
7	$B \rightarrow \neg A$	$I\rightarrow/CP$

d. 1	$\phi \rightarrow (\psi \rightarrow \chi)$	assumption
2	$\phi \wedge \psi$	assumption
3	$\phi$	$E\wedge, 2$
4	$\psi \rightarrow \chi$	$E\rightarrow/MP, 1, 3$
5	$\psi$	$E\wedge, 2$
6	$\chi$	$E\rightarrow/MP, 4, 5$
7	$(\phi \wedge \psi) \rightarrow \chi$	$I\rightarrow/CP$

e. 1	$(\phi \wedge \psi) \rightarrow \chi$	assumption
2	$\phi$	assumption
3	$\psi$	assumption
4	$\phi \wedge \psi$	$I\wedge, 2, 3$
5	$\chi$	$E\rightarrow/MP, 1, 4$
6	$\psi \rightarrow \chi$	$I\rightarrow/CP$
7	$\phi \rightarrow (\psi \rightarrow \chi)$	$I\rightarrow/CP$

f. 1	$\neg \forall x Gaxb$	assumption
2	$\forall x Gaxb$	assumption
3	$\perp$	$E\neg, 1, 2$
4	$\exists x Gbxa$	$EFSQ, 3$
5	$\forall x Gaxb \rightarrow \exists x Gbxa$	$I\rightarrow/CP$
6	$\neg \forall x Gaxb \rightarrow (\forall x Gaxb \rightarrow \exists x Gbxa)$	$I\rightarrow/CP$

## Problem 2: Partially completed longer logical arguments.

1	$\phi \wedge (\psi \vee \chi)$	assumption
2	$\phi$	$E\wedge, 1$
3	$\psi \vee \chi$	$E\wedge, 1$
4	$\psi$	assumption
5	$\phi \wedge \psi$	$I\wedge, 2, 4$
6	$\psi \rightarrow (\phi \wedge \psi)$	$I\rightarrow/CP$
7	$\chi$	assumption
8	$\phi \wedge \chi$	$I\wedge, 2, 7$
9	$\chi \rightarrow (\phi \wedge \chi)$	$I\rightarrow/CP$
10	$(\phi \wedge \psi) \vee (\phi \wedge \chi)$	$CD, 3, 6, 9$

1	$(\phi \wedge \psi) \vee (\phi \wedge \chi)$	assumption
2	$\phi \wedge \psi$	assumption
3	$\phi$	$E\wedge, 2$
4	$\psi$	$E\wedge, 2$
5	$\psi \vee \chi$	$I\vee, 4$
6	$\phi \wedge (\psi \vee \chi)$	$I\wedge, 3, 5$
7	$(\phi \wedge \psi) \rightarrow (\phi \wedge (\psi \vee \chi))$	$I\rightarrow/CP$
8	$\phi \wedge \chi$	assumption
9	$\phi$	$E\wedge, 8$
10	$\chi$	$E\wedge, 8$
11	$\psi \vee \chi$	$I\vee, 10$
12	$\phi \wedge (\psi \vee \chi)$	$I\wedge, 9, 11$
13	$(\phi \wedge \chi) \rightarrow (\phi \wedge (\psi \vee \chi))$	$I\rightarrow/CP$
14	$\phi \wedge (\psi \vee \chi)$	$E\vee, 1, 7, 13$

**Problem 3: Longer logical arguments.**

1	$\neg(\phi \wedge \psi)$	assumption
2	$\neg(\neg\phi \vee \neg\psi)$	assumption
3	$\phi$	assumption
4	$\psi$	assumption
5	$\phi \wedge \psi$	I $\wedge$ , 3, 4
6	$\perp$	E $\neg$ , 1, 5
7	$\neg\psi$	I $\neg$ /RAA
8	$\neg\phi \vee \neg\psi$	I $\vee$ , 7
9	$\perp$	E $\neg$ , 2, 8
10	$\neg\phi$	I $\neg$ /RAA
11	$\neg\phi \vee \neg\psi$	I $\vee$ , 10
12	$\perp$	E $\neg$ , 2, 11
13	$\neg\neg(\neg\phi \vee \neg\psi)$	I $\neg$ /RAA
14	$\neg\phi \vee \neg\psi$	$\neg\neg$ , 13

1	$\neg\phi \vee \neg\psi$	assumption
2	$\phi \wedge \psi$	assumption
3	$\neg\phi$	assumption
4	$\phi$	E $\wedge$ , 2
5	$\perp$	E $\neg$ , 3, 4
6	$\neg\neg\phi$	I $\neg$ /RAA
7	$\neg\psi$	DS, 1, 6
8	$\psi$	E $\wedge$ , 2
9	$\perp$	E $\neg$ , 7, 8
10	$\neg(\phi \wedge \psi)$	I $\neg$ /RAA

**Problem 4: Logical arguments using equivalence laws.**

a.

1	$A \leftrightarrow \neg B$	assumption
2	$A \rightarrow \neg B$	E $\leftrightarrow$ , 1
3	$\neg A \vee \neg B$	Impl, 2
4	$\neg B \rightarrow A$	E $\leftrightarrow$ , 1
5	$\neg\neg B \vee A$	Impl, 4
6	$(\neg\neg B \vee A) \wedge (\neg A \vee \neg B)$	I $\wedge$ , 5, 3
7	$(A \vee \neg\neg B) \wedge (\neg A \vee \neg B)$	Comm, 6
8	$(A \vee B) \wedge (\neg A \vee \neg B)$	DN, 7

e.

1	$Ma \vee Ma$	assumption
2	$Ma$	assumption
3	$Ma$	Rep, 2
4	$Ma \rightarrow Ma$	I $\rightarrow$ /CP
5	$Ma$	E $\vee$ , 1, 4, 4
6	$(Ma \vee Ma) \rightarrow Ma$	I $\rightarrow$ /CP

b.

1	$(X \wedge Y) \vee Z$	assumption
2	$\neg Y$	assumption
3	$\neg X \vee \neg Y$	I $\vee$ , 2
4	$\neg(X \wedge Y)$	DeM, 3
5	$Z$	DS, 1, 4
6	$\neg Y \rightarrow Z$	I $\rightarrow$ /CP

f.

1	$P \wedge Q$	assumption
2	$Q \wedge P$	Comm, 1
3	$(P \wedge Q) \rightarrow (Q \wedge P)$	I $\rightarrow$ /CP
4	$\neg(P \wedge Q) \vee (Q \wedge P)$	Impl, 3

c.

1	$F \rightarrow (G \rightarrow H)$	assumption
2	$F \rightarrow (\neg G \vee H)$	Impl, 1
3	$\neg F \vee (\neg G \vee H)$	Impl, 2
4	$\neg F \vee (H \vee \neg G)$	Comm, 3
5	$(H \vee \neg G) \vee \neg F$	Comm, 4

d.

1	$\exists xExa \wedge \exists xExb$	assumption
2	$\exists xExa \rightarrow \neg\exists xExb$	assumption
3	$\exists xExa$	E $\wedge$ , 1
4	$\neg\exists xExb$	E $\rightarrow$ /MP, 2, 3
5	$\exists xExb$	E $\wedge$ , 1
6	$\perp$	E $\neg$ , 4, 5
7	$\neg(\exists xExa \rightarrow \neg\exists xExb)$	I $\neg$ /RAA
8	$(\exists xExa \wedge \exists xExb) \rightarrow \neg(\exists xExa \rightarrow \neg\exists xExb)$	I $\rightarrow$ /CP