

For the logical arguments on this homework, you may use any of the following rules, but no others:

$I\wedge$	$I\vee$	$I\rightarrow/CP$	$I\neg/RAA$	$I\leftrightarrow$	Rep	$\neg\neg$	DS	CD
$E\wedge$	$E\vee$	$E\rightarrow/MP$	$E\neg$	$E\leftrightarrow$	EFSQ		HS	MT

Problem 1: Short logical arguments. Prove the following statements with logical arguments using any of the rules of natural deduction allowed for this problem (you cannot use any equivalence laws). These arguments can each be done in 6–7 lines with 1–2 subarguments, but more is okay (and less might be possible). Note that the last argument derives a tautology from no main assumptions!

- a. $P \rightarrow \neg Q, Q \vdash \neg P$
- b. $X \vee \neg Y, \neg Y \rightarrow X \vdash X$
- c. $\neg(A \wedge B) \vdash B \rightarrow \neg A$
- d. $\phi \rightarrow (\psi \rightarrow \chi) \vdash (\phi \wedge \psi) \rightarrow \chi$ (this proves part of Exp)
- e. $(\phi \wedge \psi) \rightarrow \chi \vdash \phi \rightarrow (\psi \rightarrow \chi)$ (this proves part of Exp)
- f. $\vdash \neg\forall xGaxb \rightarrow (\forall xGaxb \rightarrow \exists xGbx a)$

Problem 2: Partially completed longer logical arguments. Rewrite the following logical arguments which prove part of the distributive equivalence law (Dist). Replace all grey boxes with correct sentences and justifications, including the correct line numbers for the justifications. These arguments do not use any equivalence laws.

1	$\phi \wedge (\psi \vee \chi)$	assumption
2	ϕ	$E\wedge, 1$
3	$\psi \vee \chi$	$E\wedge, 1$
4	ψ	assumption
5	$\phi \wedge \psi$	_____
6	_____	_____
7	χ	assumption
8	$\phi \wedge \chi$	_____
9	_____	_____
10	$(\phi \wedge \psi) \vee (\phi \wedge \chi)$	_____

1	$(\phi \wedge \psi) \vee (\phi \wedge \chi)$	assumption
2	_____	assumption
3	ϕ	$E\wedge, 2$
4	_____	_____
5	$\psi \vee \chi$	_____
6	$\phi \wedge (\psi \vee \chi)$	$I\wedge, 3, 5$
7	_____	_____
8	_____	assumption
9	ϕ	$E\wedge, 8$
10	_____	_____
11	$\psi \vee \chi$	_____
12	$\phi \wedge (\psi \vee \chi)$	$I\wedge, 9, 11$
13	_____	_____
14	$\phi \wedge (\psi \vee \chi)$	_____

Problem 3: Longer logical arguments. Prove part of DeMorgan’s equivalence law (DeM) with two logical arguments, $\neg(\phi \wedge \psi) \vdash \neg\phi \vee \neg\psi$ and $\neg\phi \vee \neg\psi \vdash \neg(\phi \wedge \psi)$, using any of the rules of natural deduction allowed for this problem (you cannot use any equivalence laws). One argument can be done in 10 lines with 2 subarguments. The other argument can be done in 14 lines with 3 subarguments. More is okay (and less might be possible).

Problem 4: Logical arguments using equivalence laws. Prove the following statements with logical arguments using any of the rules of natural deduction allowed for this problem. You may also use any of the following equivalence laws, but no others:

DN	Comm	Assoc	Dist	DeM	Contra	Exp	Impl	Equiv
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These arguments can each be done in 4–8 lines with 0–2 subarguments, but more is okay (and less might be possible). Note that the last three arguments derive tautologies from no main assumptions!

- a. $A \leftrightarrow \neg B \vdash (A \vee B) \wedge (\neg A \vee \neg B)$
- b. $(X \wedge Y) \vee Z \vdash \neg Y \rightarrow Z$
- c. $F \rightarrow (G \rightarrow H) \vdash (H \vee \neg G) \vee \neg F$
- d. $\vdash (\exists xExa \wedge \exists xExb) \rightarrow \neg(\exists xExa \rightarrow \neg\exists xExb)$
- e. $\vdash (Ma \vee Ma) \rightarrow Ma$
- f. $\vdash \neg(P \wedge Q) \vee (Q \wedge P)$