LING 230 **Homework #6: Natural Deduction with Quantification** **Fall 2009**

due Monday, November 9th

For logical arguments in this homework assignment, you may use any of the following rules and equivalence laws, but no others:

I$\wedge$	I$\vee$	I$\rightarrow$/CP	I$\neg$/RAA	I$\leftrightarrow$	I$\forall$/UG	I$\exists$/EG	Rep	$\neg\neg$	DS	CD
E$\wedge$	E$\vee$	E$\rightarrow$/MP	E$\neg$	E$\leftrightarrow$	E$\forall$/UI	E$\exists$/EI	EFSQ		HS	MT
DN	Comm	Assoc	Dist	DeM	Contra	Exp	Impl	Equiv	QN	

Problem 1: Short logical arguments. Prove the following statements with logical arguments using any of the rules of natural deduction allowed for this assignment (though you should not need to use E$\exists$/EI or I$\forall$/UG). These arguments can each be done in 6–9 lines with 0–1 subarguments, but more is okay (and less might be possible). Note that the last argument derives a tautology from no main assumptions!

a. $\neg\exists x(Px \rightarrow Qx) \vdash \neg Qg$
b. $\neg\exists x\exists y(Ay \wedge Ex) \vdash \neg Aa \vee \neg Ee$
c. $\forall x\forall y Rxy \vdash \exists y\exists x(Ryx \wedge Rxy)$
d. $\vdash \exists z(\neg Gz \vee Gz)$

Problem 2: Partially completed longer logical arguments. Rewrite the following logical arguments which prove the quantifier negation equivalence law (QN) for the special case in which ϕ is Px. Replace all grey boxes with correct sentences and justifications, making sure to fill in the correct line numbers in the justifications, using any of the rules and equivalence laws of natural deduction allowed for this assignment (except for QN itself, of course!)

1	$\neg\exists xPx$	assumption
2		assumption
3		
4		
5	$\neg Pa$	I$\neg$/RAA
6	$\forall x\neg Px$	

1	$\forall x\neg Px$	assumption
2		assumption
3	Pa	
4		E$\forall$/UI, 1
5		E$\neg$, 4, 3
6		
7	$Pa \rightarrow \neg\forall x\neg Px$	
8		E$\exists$/EI, 2, 7
9	$\bot$	
10	$\neg\exists xPx$	

1	$\neg\forall xPx$	assumption
2		assumption
3		assumption
4	$\exists x\neg Px$	I$\exists$/EG, 3
5		E$\neg$, 2, 4
6	$\neg\neg Pa$	
7	Pa	DN, 6
8	$\forall xPx$	
9	$\bot$	E$\neg$, 1, 8
10		I$\neg$/RAA
11	$\exists x\neg Px$	DN, 10

1	$\exists x\neg Px$	assumption
2	$\neg Pa$	assumption
3		assumption
4		E$\forall$/UI, 3
5	$\bot$	E$\neg$, 2, 4
6		
7		
8	$\neg\forall xPx$	E$\exists$/EI, 1, 7

Problem 3: Longer longer logical arguments. Prove the following statements with logical arguments using any of the rules and equivalence laws of natural deduction allowed for this assignment. These arguments can each be done in 10–13 lines with 0–3 subarguments, but more is okay (and less might be possible). Note that the last argument derives a tautology from no main assumptions!

a. $\forall x(Ax \rightarrow Bx), \forall x(Bx \rightarrow Cx), \forall xAx \vdash \forall x(Bx \wedge Cx)$
b. $\forall x(Ax \rightarrow Bx) \vdash \forall x\forall y\big((Ax \wedge Ay) \rightarrow (Bx \wedge By)\big)$
c. $\exists x(Px \vee Qx) \vdash \exists yPy \vee \exists zQz$
d. $\vdash \forall z(\neg Gz \vee Gz)$